cidal attributes of this chemical have already been established for cotton and various legumes.

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[1] Eaton, F. M., *Science*, **126**, 1174 (1957).
[2] Moore, J. F., *Science*, **129**, 1738 (1959).
[3] U.S. Patents: 2,889,662 (1959) "Production of Hybrid Cottonseed by Chemically Inducing Male Sterility"; 2,948,987 (1960) "Selective Gametocidal Process"; 2,974,444 (1961) "Inducing Male Sterility in Tomatoes"; 2,994,599 (1961) "Regulating Plant Growth".
[4] Moore, J. F., *Proc. Amer. Soc. Hort. Sci.*, **84**, 474 (1964).

GENERAL

Kinematic Programming for Rain

THE following statement and resolution of an elementary optimization problem may be of interest to those who, like me, have never been confident about the optimal speed and posture to adopt when caught without protection in a rain shower.

Consider a bounded rigid body B of arbitrary shape in a Euclidean space of k dimensions ($k > 1$). B is required to move one of its points, a reference point O, say, from a starting point S to a finishing point F. The space is dotted with hostile points generically denoted by the symbol R, which are all moving with the same velocity $\mathbf{v}$. It is B's object to move O from S to F in such a way as to minimize his contact with R.

An *itinerary* between S and F is specified by the triad $I = [P_0, \omega(t), \omega(t)]$, where P_0 is the initial posture of B at S, $\mathbf{u}(t)$ is the velocity of O at time t and $\omega(t)$ is the angular velocity vector. Let $\mathfrak{I}$ denote the class of available I's. Assume that, if a portion of the surface of B sweeps out a volume δV of the space, the mean contact with R is proportional to δV if δV, regarded as part of the moving space (R-space say) in which R is stationary, has not been previously swept by B. Also that, once a volume in R-space has been swept, that volume remains free of R thereafter. The problem is therefore to minimize:

$$V = \int_{(S)}^{(F)} \frac{dV}{dt}\, dt \qquad (1)$$

where dV/dt is the rate of sweeping by the whole surface of B of previously unswept volume. Now:

$$\frac{dV}{dt} = \int \varphi(\mathbf{r},t)\,[\mathbf{u}(t) - \mathbf{v} + \omega(t) \wedge \mathbf{r}]\,.\,d\mathbf{S} \qquad (2)$$

where $d\mathbf{S}$ is the normal vector surface element of B at the point $\mathbf{r}$ relative to O and $\varphi(\mathbf{r},t) = 1$ if the rest of the integrand is not negative and $\mathbf{r}$ is adjacent to previously unswept volume: $\varphi(\mathbf{r},t) = 0$ otherwise. Equations (1) and (2) combine to give V for any itinerary.

Fortunately, the expected is the case. As long as $\mathfrak{I}$ includes "top-speed, unrestricted posture, straight-line" itineraries along SF, then choice of I to minimize V can be made using elementary considerations. Let:

$$u_m = \max_{I \varepsilon \mathfrak{I}} \max_{t} u(t)$$

and let $V_t(\mathbf{p})$ be the volume in k–1 dimensions of the projection of B at time t parallel to $\mathbf{p}$ on to the k–1 space orthogonal to $\mathbf{p}$. Let min $V_t(\mathbf{p}) = M$ (say) $= V_t(\mathbf{p}_t)$ (say). Let $\mathbf{l}$ be the unit vector along $\overrightarrow{SF}$.

Case A. If $\mathbf{v}\,.\,\mathbf{l} \leqslant 0$, then an optimal itinerary in $\mathfrak{I}$ is $I_m = [P'_0, u_m\,\mathbf{l}, \mathbf{o}]$, where P'_0 has $\mathbf{p}_0$ parallel to $\mathbf{v} - u_m\,\mathbf{l}$. To see this, Fig. 1 in R-space should be examined.

S_0, F_0 are the positions of S and F, respectively, at $t = 0$. F_T is the position of F at $t = T$ if O arrives at F_T along I_m. F_T' is the position of F at $t = T'$ if O arrives at F_T' using the arbitrary(dashed) itinerary I_A. Clearly I_m minimizes the time of the itinerary, whence $T \leqslant T'$ and $S_0F_T \leqslant S_0F_T'$ as shown. Thus, using I_A, B must sweep out a volume of R-space which cannot be less than the volume for I_m. [The degenerate case of $\mathbf{v}$ parallel to $\mathbf{l}$ does not affect this finding.]

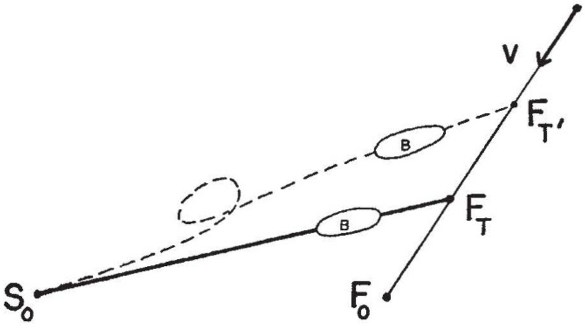

Fig. 1. Straight line and arbitrary itineraries in R-space

Case B. If $\mathbf{v}\,.\,\mathbf{l} > 0$, then an optimal itinerary is: (i) I_m if $u_m \leqslant v^2/\mathbf{v}\,.\,\mathbf{l}$; (ii) $I_0 = [P''_0, (v^2/\mathbf{v}\,.\,\mathbf{l})\,\mathbf{l}, \mathbf{o}]$, where P''_0 has $\mathbf{p}_0$ parallel to $\mathbf{v} - (v^2/\mathbf{v}\,.\,\mathbf{l})\,\mathbf{l}$, if $u_m > v^2/\mathbf{v}\,.\,\mathbf{l}$. Case B (i) is proved in the same way as case A, because, for $u_m \leqslant v^2/\mathbf{v}\,.\,\mathbf{l}$, an essentially similar figure is obtained. For case B (ii), S_0F_T is perpendicular to F_0F_T for I_0, which also establishes the result.

In cases A and B (i), the optimal itinerary is seen to be unique. Any deviation from it will increase V. However, in case B (ii) there is non-uniqueness. This is readily seen for the extreme case when $u_m = \infty$, when the solid itinerary indicated in original space in Fig. 2 has the same V as I_0.

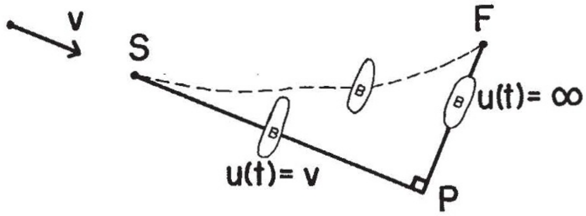

Fig. 2. Non-uniqueness

For $v^2/\mathbf{v}\,.\,\mathbf{l} < u_m < \infty$, there are itineraries with O-paths interior to the triangle SPF of Fig. 2, as dotted, their characteristic being increasing distances from SP and a resolved velocity parallel to SP equal to $\mathbf{v}$.

The practical recommendations arising from these considerations are that, when rain is coming from the general direction of your destination, you should get your head down and go as fast as you can. When the rain is coming from behind, either lean backwards or walk backwards while leaning 'forwards', in neither case exceeding a certain optimal speed.

When the choice of posture is unrestricted, the only property of B affecting min V is the minimum projection M. When posture is restricted so that other properties of B affect min V, the following fact may be useful. For a given itinerary, V cannot be increased and, especially when posture is restricted, may well be decreased by filling in any concavities in B's surface with portions cut off anywhere from B.

The possibility of reducing min V by cutting B into portions and sending these portions separately to their assembly points is always available in certain applications (but not in that originally envisaged). Consider B shaped like a letter L. Break it at the angle. The flexibility thereby introduced can be used to reduce min V. However, care is needed because if B were cut into portions similar to B, it is easy to see from scalar considerations that min V would increase.

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